Hungarian

JJV

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Reminder

Potential ℓ does not need to be positive

Only constraint: $\forall i, j, \ell(i) + \ell(j) \leq W_{ij}$.

$O(N^4)$ algorithm

Initialize $\ell = 0$

Consider the subgraph of tight edges: $\{(i,j) \mid \ell(i) + \ell(j) = W_{ij}\}$

Search path from unmatched nodes left. Z is set of reachable nodes.

If path reaches unmatched node right: we found a bigger matching M'.

Otherwise: path "finishes" on the left. Consider the frontier:

$$\Delta = \min_{\substack{i \in Z \cap U \\ j \in V \setminus Z}} \{W_{ij} - \ell(i) - \ell(j)\}$$

Increase by Δ on the left, decrease by Δ on the right. Lemma: current matching M stays tight (so M stays in the subgraph of tight edges)

Loop invariant

Either:

- M is maximum size: algorithm is over
- subgraph contains an augmenting path
- graph contains a loose-tailed path: path from unmatched node left to some node w_{next} on the right s.t. all edges are tight except the last one.

Useful lemmas

$$\sum_{i \in U} \ell(i) + \sum_{j \in V} \ell(j) \leq \text{any cost of matching}$$

- Invariant: ℓ stays a potential: $\forall i, j, \ell(i) + \ell(j) \leq W_{ij}$
- Adjusting potential leaves M unchanged
- After each potential recalculation, Z grows
- At most n potential recalculations happen before an augmenting path is found (because we discover at most n new nodes on the right).

Grey areas

- Why a vertex cover is good for $O(N^4)$ algorithm? (TopCoder)
- How to recompute Δ and next right node w_{next} in $O(n)$? (Wikipedia, CP Algorithm)

Nice plots: Codeforces post